Duration Dependence in US Expansions: A re-examination of the evidence

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It is commonly accepted that economic expansions do not exhibit duration dependence, that is, the probability of an expansion terminating in the near future is thought to be independent of the length of the expansion. The most cited recent evidence in support of this claim is based on the work of Rudebusch [2016], which builds on Diebold and Rudebusch [1990] and Diebold, Rudebusch, and Sichel [1993]. In this short note, we re-examine this claim. Our main focus is on determining the probability of the US economy entering a recession in the following year (or following two years) conditional on the expansion having lasted q quarters. This contrasts only slightly with Rudenbusch's focus which examines the probability of entering into a recession *next month* conditional on being in an expansion that has lasted p months. When looking at the probability of entering a recession within a year (or 2 years), we find considerable evidence of economically significant duration dependence, especially when adopting a non-parametric approach. For example, for an expansion that has lasted only 5 quarters, the probability of entering a recession in the next year is around 10%, while this increases to 30-40% if the expansion has lasted over 35 quarters. Similarly, if looking at a two years window, we find the probability of entering a recession in the next two years raises from 25-30% to around 50-80% as the expansion extends from 5 quarters to 32 quarters.[1] This pattern suggests that certain types of macroeconomic vulnerabilities may be accumulating as the expansion ages causing the arrival of a recession to become more likely. Our non-parametric estimates suggest that this later pattern is especially important once a recession has lasted more than 6 years.

[1] The exact probability depends on whether we use a parametric or a non parametric approach.

1 Parametric Approach

Our starting point is Rudebusch's [2016] claim that US expansion do not exhibit any duration dependence. That claim is illustrate in panel (a) of Figure 1, that replicates Rudebusch's [2016] with updated data. This Figure is obtained by estimating a Weibull survival function on US prewar and postwar data, and then using it to compute those probabilities. The data underlying this figures correspond to the series of zeros and ones associated with expansions and recessions as recorded by the NBER. The data frequency is monthly, and the unfinished recession at the end of each of the two samples is treated as right-censored observations. The finding emphasized by Rudebusch is that the dark gray line, which relates to the post-war sample, is very flat. If there were no duration dependence, then this curve should be perfectly flat. Since it is quite flat, it is commonly inferred that this supports little economically significant duration dependence. From this estimation one finds that the estimated probability of a recession starting next month is 4% if the expansion is aged 10 quarters, 6% if aged 20, 8% if aged 30 and 9% if aged 40.

As we will be focusing on quarterly data in the rest of this note, panel (b) of Figure 1 redoes Rudebusch's analysis using quarterly data. This Figures shows that, when estimated with quarterly data instead of monthly data, the parametric approach based on the Weibull distribution delivers almost identical results.[2]

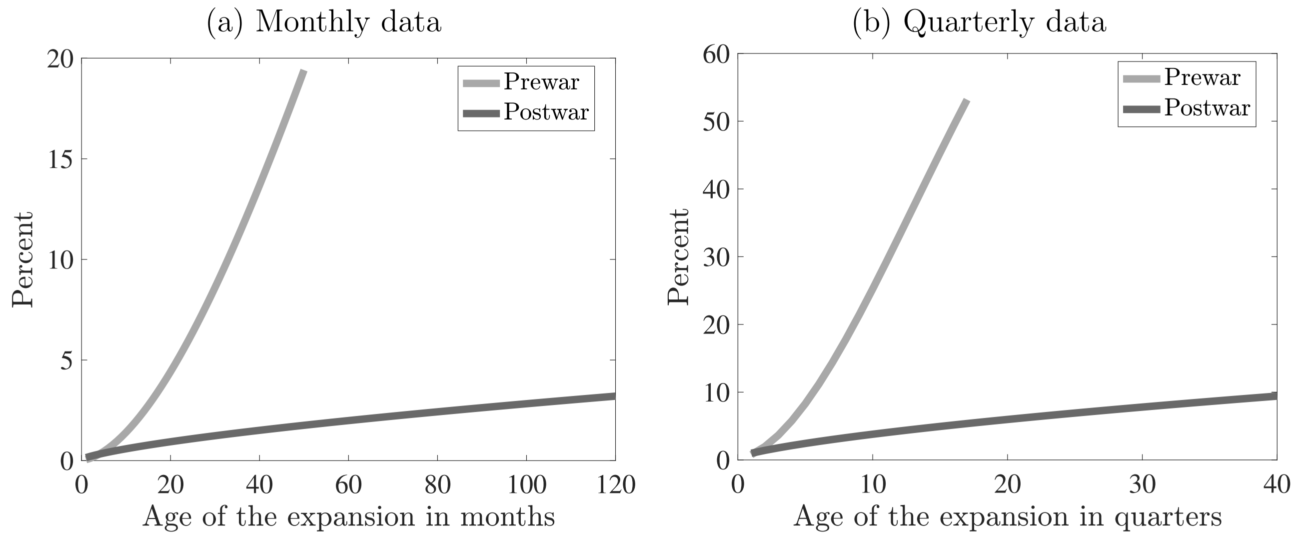
Note that our estimate of the shape parameter of the Weibull for the postwar period, based on quarterly data is 1.70 with a 95% confidence interval [1.05 2.75]. The constant hazard configuration (when the Weibull is an exponential distribution) corresponds to a value of 1. At the 95% level, this excludes the absence of age dependance, but that dependance seems minor when looking at the probability of a recession starting in the next quarter.

Let us now look over a one-year or two-years horizon, *i.e.*focus on the probability of the expansion ending within the next four or next eight quarters. This is computed given the estimated parameters of the Weibull, and the result is shown in Figure 2.

Although the Weibull parameters are the same as in Figure 1, the positive slope is more visible. In postwar data, the probability of a recession in the next year is 16% if the expansion is aged 10 quarters, 23% if aged 20, 29% if aged 30 and 33% if aged 40. The probability of a recession in the next two years is 31% if the expansion is aged 10 quarters, 42% if aged 20, 50% if aged 30, 57% if aged 40.

[2] Note that with quarterly data, falling into a recession in the next quarter corresponds to falling into a recession in the next three month, so that the probabilities numbers in panel (b) are roughly three times those of panel (a).

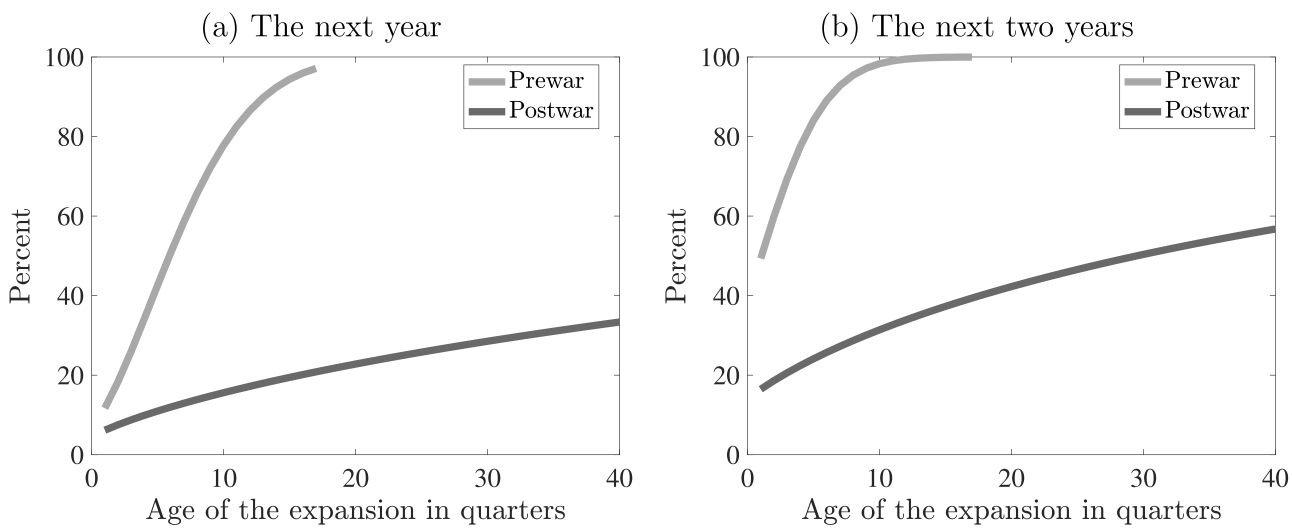
Figure 1: Probability of an expansion ending – next month or quarter – as a function of its age



Notes: These probabilities are computed from the estimated Weibull survival function. Estimation is done using monthly (panel (a)) or quarterly (panel (b)) NBER datation for expansions and recessions. Prewar sample is October 1854 to January 1941, postwar is September 1945 to January 2019.

Our main point here is that, even if we restrict attention to the parametric formulation adopted in Rudebusch [2016], looking at the probability of entering into a recession within the next year or two – instead of during the next month –can change one's perspective. When looking at yearly hazard rates, one is much more inclined to conclude that the degree of duration dependence in the data is economically significant.

Figure 2: Probability of an expansion ending in the next year or the next two years



Notes: These probabilities are computed from the estimated Weibull survival function. Estimation is done quarterly NBER datation for expansions and recessions. Prewar sample is October 1854 to January 1941, postwar is September 1945 to January 2019.

2 Non-Parametric Approach

An alternative approach to the question at hand is to look at this issue using a less parametric approach. In particular, the Weibull parameterization assumes that the survival rate is a monotonic function of time, which is a very strict restriction.[3] Instead, we can use the Kaplan and Meier's [1958] non-parametric estimator of the survival function, allowing for right censored data. From this survival function, we can compute the probability of being in a recession for the first time in at most p quarters, given that the age of the expansion is q quarters. We denote such a probability by $\Phi(q, p)$, where Φ is a 40×39 matrix for postwar data, and it is a 17×16 matrix for the prewar period. The two Φ matrices are shown in Figure 3.

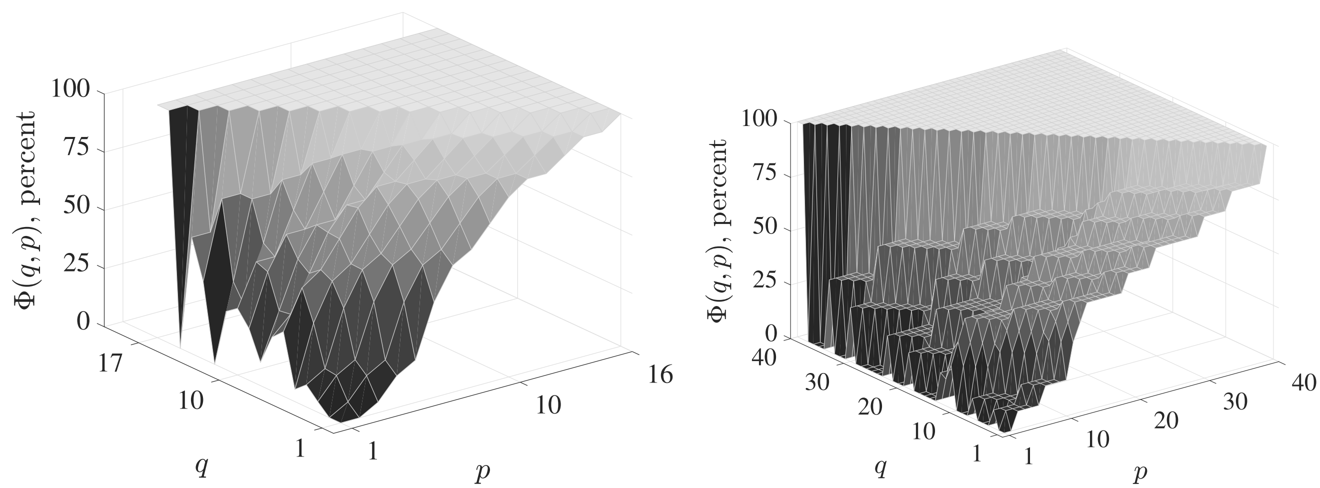
By taking a slice of the matrix Φ, one can plot the non-parametric estimate of the probability of entering in a recession in one quarter, one year or two years. As an example, this is shown in Figure 4 for one quarter, one year and two years, where the solid lines are smooth estimates of the non-parametric estimates.[4] Two observations can be made: *(i)* the parametric and non-parametric estimates are quite similar when looking at the one quarter

[3]Zuehlke [2003] explores this issue using a Mudholkar estimator, that is a parametric approach that does not impose monotonic survival rate. He finds evidence in favor of duration dependence.

[4]We use a locally weighted scatter plot smoothing method with linear regression

Figure 3: Probability $\Phi(q,p)$ of being in a recession for the first time in at most p quarters, given that the expansion age is q quarters

(a) Prewar (b) Postwar

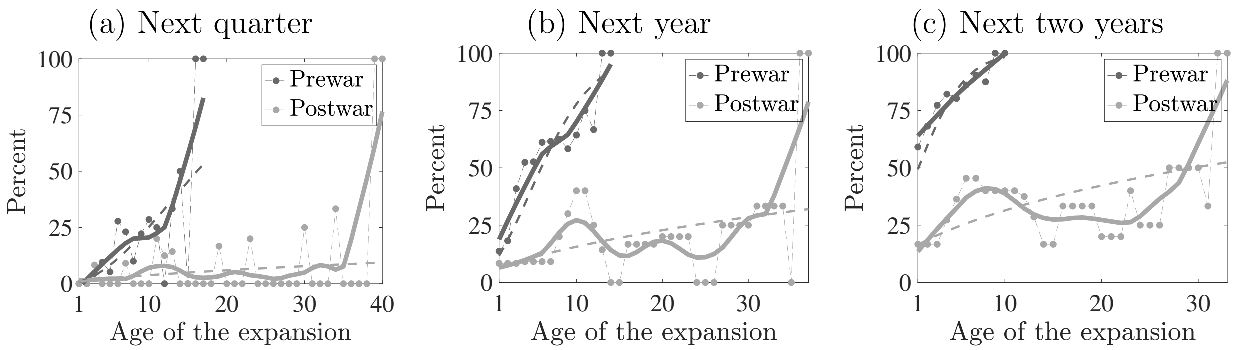


Notes: These probabilities are computed from the Kaplan and Meier's [1958] non-parametric estimator of the survival function. Estimation is done using quarterly NBER datation for expansions and recessions. Prewar sample is October 1854 to January 1941, postwar is September 1945 to January 2019.

ahead probability of entering a recession. However, the parametric and non-parametric estimates tend to diverge when looking further ahead. *(ii)* for one year and two years ahead probability of a recession, the non-parametric function is quite flat for recessions aged 10 to 25 quarters, but it becomes quite steep after 25 quarters; in other words, the probability of a recession in the next one or two years increases substantially as it ages, once the expansion is more than 25 quarters of age).

Finally, let us focus more on the detail of the 1, 1.5 and 2 years ahead recessions probabilities implied by the non-parametric approach and contrast them with the parametric estimates. To see this most clearly, in Figure 5 we plot the probability of an expansion ending the next year , the next year and a half or the next two years using only the postwar sample. The dotted lines are the parametric estimates. The thick lines are a smooth version of the non-parametric estimates. As can be seen, the parametric estimates and the smoothed non-parametric estimates give somewhat different views regarding the pattern of duration dependence, with non-parametric estimates offering a more complex narrative. The non-parametric estimates suggest that duration dependence is minimal for expansion lasting up to 25 quarters. But after 25 quarters, the duration dependence implied by the non-parametric

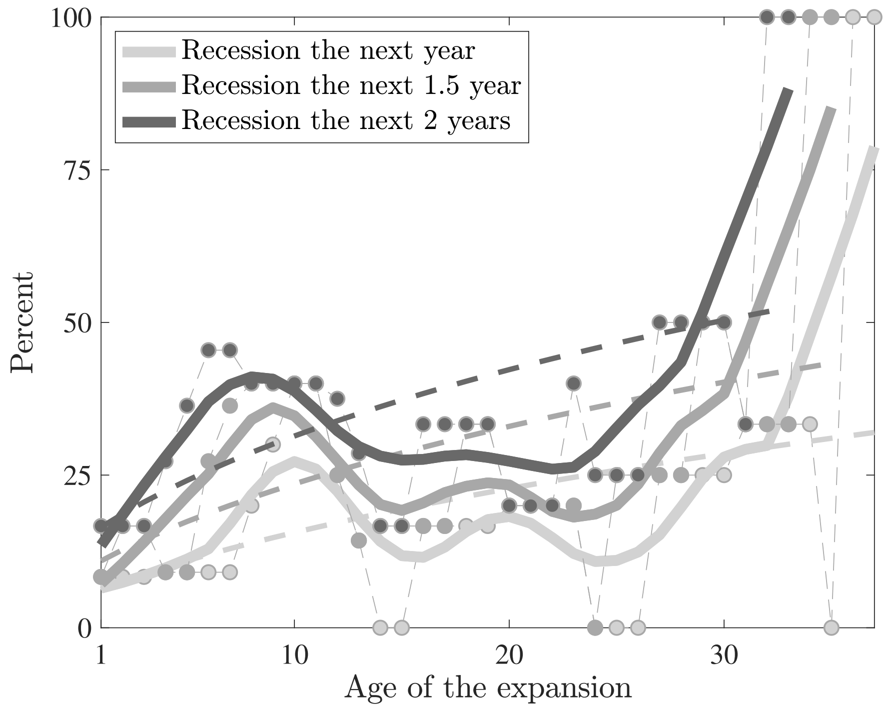
Figure 4: Probability of an expansion ending in the next quarter or the next year (parametric and non parametric hazard)



Notes: the dots are the non parametric estimates (the matrix Φ). The thick lines are smoothed version of the dots. The dashed line is the hazard function as obtained from estimating a Weibull distribution. Estimation is done using quarterly NBER datation for expansions and recessions. Prewar sample is October 1854 to January 1941, postwar is September 1945 to January 2019. The age of the expansion is in quarters.

estimates becomes very apparent. For example, when an expansion ages from 6 years to 9 years, the non-parametric estimates suggest that the probability of a recession within a year almost triples. If one looks in more detail at the initial phase of an expansion– up to 8 quarters– there is also some evidence of positive duration dependence reflecting the possible occurence of double-dip recessions. Then from 8 to 25 quarters, there appears instead to be negative duration dependence as the expansion takes hold, that is, during this phase the probability of entering a recession appears to decrease as the expansion ages. Finally, after 25 quarters the probability of entering a recession increases rapidly as the expansion gets old. This suggests that expansions may be favoring the growth of certain vulnerabilities when they are older than 6 years.

Figure 5: Probability of an expansion ending in the next quarter, the next year of the next two years, postwar sample (parametric and non parametric approach)



Notes: the dots are the non parametric estimates (the matrix Φ). The thick lines are smoothed version of the dots. The dashed line is the hazard function as obtained from estimating a Weibull distribution. Estimation is done using quarterly NBER datation for expansions and recessions for the postwar sample (September 1945 to January 2019). The age of the expansion is in quarters.

3 Conclusion

In this short note we provided updated parametric and new non-parametric estimates of the duration dependence of US expansion. Our main finding is that for expansions that have lasted more than 25 quarters, there is substantial evidence of positive duration dependence. In other words, once an expansion has lated more than 6 years, it may be favoring the growth of certain vulnerabilities that may make the onset of a recession more likely. [5] Obviously, we recognize that all our calculations are based on a small sample of data since recessions are rather rare. Given this limited data, one can do no better than make the best inference possible.

[5] Further evidence that recessions do exhibit duration dependence, see Beaudry, Galizia, and Portier [2016].

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Appendix

A Data

data are "NBER based Recession Indicators for the United States from the Period following the Peak through the Trough", as obtained from FRED. Mnemonics are `USREC` for monthly data and `USRECQ` for quarterly ones. Prewar sample is October 1854 to January 1941, postwar is September 1945 to January 2019.

B Parametric Estimates

Let x be the time to the recession. If x is distributed according to a Weibull distribution, density f is given by

$$f(x; \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}.$$

Let $F(x)$ be the cumulative density function and $S(x) = 1 - F(x)$ be the survival function.

Hazard rate is given by $f(t)/S(t)$. We compute the hazard function $h(q, p)$ which is probability of entering into a recession within the next p periods when the age of the expansion is q. the function h is given by

$$h(q, p) = \frac{F(q + p) - F(q)}{S(q)}.$$

Note that the hasard rate function is often defined as the "instantaneous" one

$$h(q) = \frac{f(q)}{S(q)}$$

This is numerically very close to $h(q, 1)$. Results from estimation of the Weibull distribution parameters are given in Table 1. Note that $k = 1$ corresponds to a constant hazard function, while the hazard function is increasing for $k > 1$. Both the pre- and postwar sample end with an unfinished expansion. We estimate the parameters with either (1) excluding the last unfinished expansion or (2) treating it as a right-censored observation. This does not make a difference for the pre-war sample, but it slightly decreases estimated k as the current (unfinished) expansion is the second longest one in postwar data. Note that in all the cases, the 95% confidence interval of k does not includes 1.

Table 1: Estimation of the Weibull distribution parameters

	Quarterly data			
	(1)		(2)	
	λ	k	λ	k
Prewar	10.06	2.86	10.28	2.88
	[8.58 11.79]	[2.08 3.93]	[8.80 12.02]	[2.09 3.97]
Postwar	21.87	1.82	24.39	1.70
	[15.53 30.78]	[1.14 2.91]	[17.06 34.88]	[1.05 2.75]
	Montly data			
	(1)		(2)	
	λ	k	λ	k
Pre-war	29.83	2.71	30.45	2.73
	[25.23 35.27]	[1.96 3.73]	[25.86 35.87]	[1.98 3.78]
Post-war	65.91	1.84	73.64	1.70
	[46.95 92.53]	[1.15 2.93]	[51.54 105.22]	[1.05 2.75]

Notes: (1) corresponds to the estimation in which the last unfinished expansion is excluded, while (2) performs the estimation considering it as a right-censored observation. Below each coefficient is the 95% confidence interval.

C Non Parametric Estimates

The Kaplan and Meier [1958] estimator of the survival function $S(t)$ (the probability that the expansion lasts longer than t) is given by:

$$\widehat{S}(t) = \prod_{i:\ t_i \leq t} \left(1 - \frac{d_i}{n_i}\right),$$

with t_i a time when at least one recession happened, d_i the number of recessions that happened at time t_i and n_i the number of expansions known to last (have not yet finished or been censored) at time t_i. From this estimated survival function, we compute the conditional probability of the expansion continuing further t periods, given that it already lasted s periods. This conditional probability, usually referred to as conditional survival, is given as

$$CS(t|s) = \frac{\widehat{S}(s+t)}{\widehat{S}(s)}.$$

The probability of a recession occurring in the next p periods conditional on the expansion being of age q is then given by

$$\Phi(q, p) = 1 - CS(p|q).$$