Does it Matter to Assume that U.S. Monetary Authorities Follow a Taylor Rule?

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Motivation
An introductory example

- Sticky prices models do not have much macro predictions *per se*.
- What maters is the bundle (sticky prices + monetary policy)
- I want first to illustrate that two "Taylor rules" with very similar policy narrative lead to very different outcomes.

Motivation
An introductory example

- Basic 3-equation New-Keynesian model

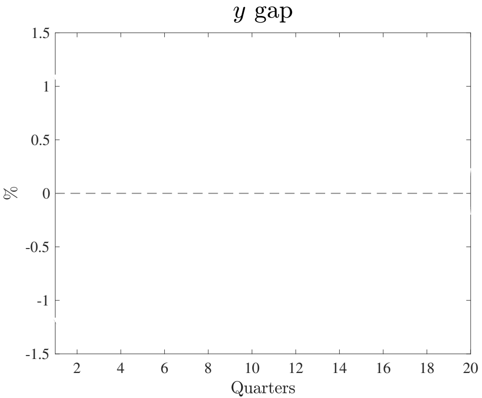
$$y_t = E_t[y_{t+1}] - (i_t - E_t[\pi_{t+1}]) + d_t, \qquad \text{(Euler Equation)}$$
$$\pi_t = .99\ E_t[\pi_{t+1}] + .1\ y_t + \mu_t, \qquad \text{(Phillips Curve)}$$

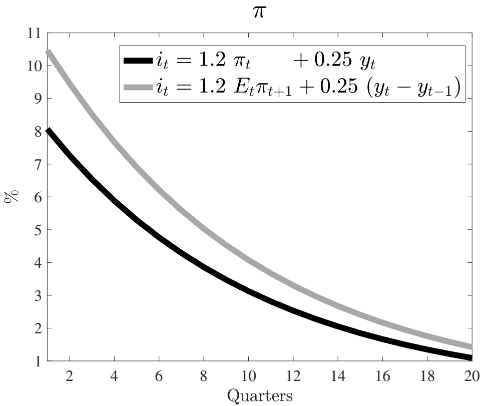
- Take two incarnations of a "Taylor rule" (similar narrative but different specification):

$$\begin{aligned} i_t &= 1.2\ \pi_t &+\ & 0.25\ y_t \\ i_t &= 1.2\ E_t\pi_{t+1} &+\ & 0.25\ (y_t - y_{t-1}) \end{aligned}$$

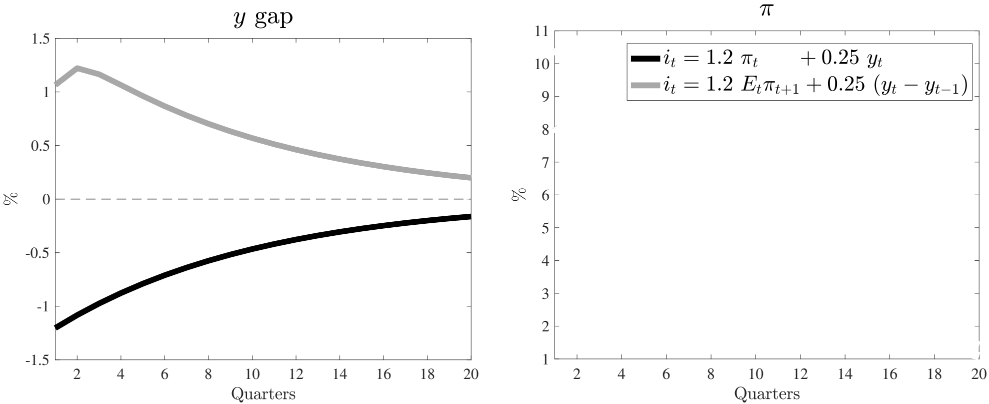
- Response to a *simultaneous* size 2 demand shock and size 1 markup shock, with persistence .9 and .9;
  - × amplitude of the response
  - × sign of the response
  - × co-movements

Response to a *simultaneous* size 2 demand shock and size 1 markup shock



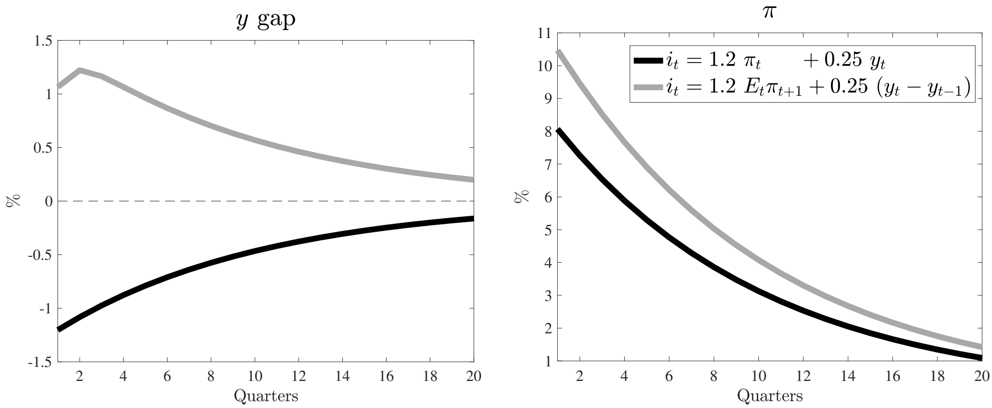


Response to a 2 s.d. demand shock *and* a 1 s.d. markup shock



Motivation
An introductory example

Response to a 2 s.d. demand shock *and* a 1 s.d. markup shock



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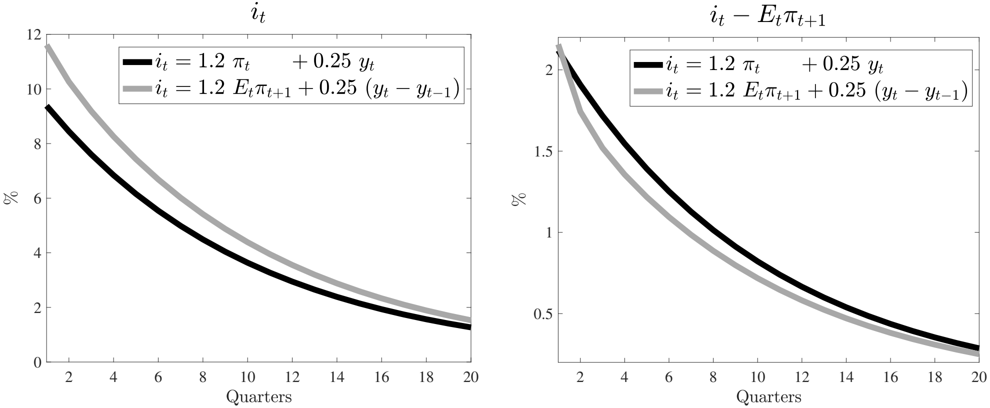

- Taylor rule type policy = assuming some very specific constraints to the conduct of monetary policy.
- Do we believe that a Taylor rule describes *realistically* the precise constraints that monetary authorities face?
- There is a surprising discrepancy between the amount of effort put in microfounding DSGE models and the lack of justification of the Taylor rule specification (*as if it should not matter*).

What we do?

- We explore the consequences of assuming (possibly wrongly) that monetary authorities follow a Taylor rule when estimating NK models.
- We promote the specification of (linear) *state-dependent* monetary policy rules, that maps the state of the economy into an equilibrium allocation.

Literature

- There is a literature on optimal monetary policy – not our concern here.
- The DSGE literature is being very creative on how to specify “the” Taylor rule, with no micro-foundations.
- Cúrdia, Ferrero, Ng and Tambalotti [2015] do estimations with various policy rules, with a focus on how well those rules track the nominal interest rate.

Important remark

- In the applied macro literature, virtually no one estimates indeterminate models
  - possibly a wrong choice, but not discussed here
- In what we do, estimation will always be restricted to the set of parameters that gives determinacy
- This is for example what `Dynare` is doing behind the scene
- In other words, one identifying restriction is equilibrium determinacy.

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

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2. A Simple Three-Equation Estimated Model
3. Extensions

1. Abstract Approach

- Goal: show the existence of a determinacy bias and a misspecification bias
- Start with the determinacy bias

1. Abstract Approach

- y is the variable of interest ("output")

$$y_t = \alpha E_t y_{t+1} + \beta i_t + s_t, \qquad 0 \leq \alpha < 1 \text{ and } \beta > 0.$$

- s is an autoregressive shock

$$s_t = \rho s_{t-1} + \varepsilon_t, \qquad 0 \leq \rho < 1 \text{ and } V(\varepsilon) = 1$$

- i is a policy variable ("interest rate") that helps controlling y
  - × feedback rule (ϕ for ϕ*eedback*) ("TAYLOR rule")

$$i_t = \phi y_t, \tag{1}$$

  - × state-dependent rule (σ for σ*tate-dependent*)

$$i_t = \sigma s_t. \tag{2}$$

1. Abstract Approach

Solution with a feedback rule $\quad i_t = \phi y_t$

- $y_t = \alpha E_t y_{t+1} + \beta \underbrace{i_t}_{\phi y_t} + s_t$
- Solving forward

$$y_t = \frac{1}{1-\beta\phi}\left(\sum_{j=0}^{\infty}\left(\frac{\alpha\rho}{1-\beta\phi}\right)^j\right)s_t$$

- The condition for this sum to converge for any persistence parameter $0 \leq \rho < 1$ is $\left|\frac{\alpha}{1-\beta\phi}\right| < 1$.
- Therefore, the restriction on policy to have determinacy ("Taylor principle") is

$$\phi \notin \left]\frac{1-\alpha}{\beta}, \frac{1+\alpha}{\beta}\right[$$

- and solution is

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\,s_t$$

1. Abstract Approach

Solution with a state-dependent rule $i_t = \sigma s_t$

- $y_t = \alpha E_t y_{t+1} + \beta \underbrace{i_t}_{\sigma s_t} + s_t$
- Solving forward

$$y_t = (1 + \beta\sigma) \left(\sum_{j=0}^{\infty} (\alpha\rho)^j \right) s_t$$

- The sum converges for *any* policy choice σ
- and solution is

$$y_t = \frac{1 + \beta\sigma}{1 - \alpha\rho} s_t$$

Feedback rule

$$i_t = \phi \, y_t$$

Solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho} \, s_t$$

State-dependent rule

$$i_t = \sigma \, s_t$$

Solution

$$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \, s_t$$

- Is that irrelevant to specify one rule or the other?
- Can we always find a σ (a state-dependent rule) that produces the same allocations than a ϕ (a feedback rule) and reciprocally?

DGP = Feedback rule

$$i_t = \phi \, y_t$$

Solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho} \, s_t$$

State-dependent rule

$$i_t = \sigma \, s_t$$

Solution

$$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \, s_t$$

- **Equivalent state-dependent rule:** Consider a feedback rule model with parameter ϕ such that the solution is determinate. A state-dependent rule with parameter σ^E generates the same allocations *if and only if*
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$$\frac{1}{1-\beta\phi-\alpha\rho} = \frac{1+\beta\sigma}{1-\alpha\rho} \iff \sigma^E = \frac{\phi}{1-\beta\phi-\alpha\rho}.$$

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- *Any feedback rule policy allocations can be replicated by a state-dependent policy.*

Feedback rule	**DGP = State-dependent rule**
$i_t = \phi\ y_t$	$i_t = \sigma\ s_t$
Solution	**Solution**
$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\ s_t$	$y_t = \frac{1+\beta\sigma}{1-\alpha\rho}\ s_t$

- **Equivalent feedback rule:** Consider a state-dependent rule model with parameter σ. A *necessary condition* for a feedback rule with parameter ϕ^E to generates the same allocations is that

  ×

  ×

Feedback rule

$$i_t = \phi \; y_t$$

Solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho} \; s_t$$

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×

×

Feedback rule	**DGP = State-dependent rule**
$i_t = \phi \; y_t$	$i_t = \sigma \; s_t$
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×

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  - × If $\sigma > -\frac{1+\alpha}{2\alpha\beta}$, then ϕ^E will satisfy the determinacy condition.
  - × But if $\sigma < -\frac{1+\alpha}{2\alpha\beta}$, there is no determinate feedback model that can reproduce the state-dependent rule allocations.

- Feedback rules are likely to lead to bias in estimation
- This can be shown analytically in our abstract model

- We start with an extreme example for which the bias can be analytically computed.
- Only y is observed.
- All parameters are known but α: $\beta,\ \rho,\ \phi$ or σ, ϕ^E or σ^E

$$y_t = \alpha E_t y_{t+1} + \beta i_t + s_t$$

- α can be estimated (ML) by matching the observed variance of y: V_y

- If the econometrician knows the DGP (feedback or state-dependent rule), α is estimated without bias.

Feedback rule solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\, s_t$$

$$V_y = V^{\phi}(\alpha) = \frac{1}{(1-\beta\phi-\alpha\rho)^2}\,\frac{1}{1-\rho^2}$$

$$V^{\phi}(\widehat{\alpha}) = \frac{1}{(1-\beta\phi-\widehat{\alpha}\rho)^2}\,\frac{1}{1-\rho^2}$$

$$\rightsquigarrow \quad \widehat{\alpha} = \begin{cases} \alpha \\ -\alpha + \frac{2}{\rho}(1-\beta\phi) \end{cases}$$

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State-dependent rule solution

$$y_t = \frac{1+\beta\sigma}{1-\alpha\rho}\, s_t$$

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$$\rightsquigarrow \quad \widehat{\alpha} = \begin{cases} \boldsymbol{\alpha} \\ -\alpha + \frac{2}{\rho} \end{cases}$$

- What if the econometrician is wrong about the policy rule (feedback or state-dependent)?

- **Assume the DGP is the feedback rule** (ϕ) but the econometrician believes it is a state-dependent rule model with $\sigma = \sigma^E$.
- An estimator of α is then given by $\widehat{\alpha}$ that solves

$$\underbrace{V^{\sigma}(\widehat{\alpha})}_{\left(\frac{1+\beta\sigma^E}{1-\widehat{\alpha}\rho}\right)^2 \frac{1}{1-\rho^2}} = \underbrace{V_y}_{\text{data}} = \underbrace{V^{\phi}(\alpha)}_{\text{DGP}}$$

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1. Abstract Approach

Possible Bias when Estimating α

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- Two solutions : α and $-\alpha + \frac{2}{\rho} > 1$.
- Impose determinacy $\rightsquigarrow \widehat{\alpha} = \alpha$
- All good!
- *It does not hurt to assume a state-dependent rule if the true model has a feedback rule*

- **Assume the DGP is the state-dependent rule** (σ) but the econometrician believes it is a feedback rule model.
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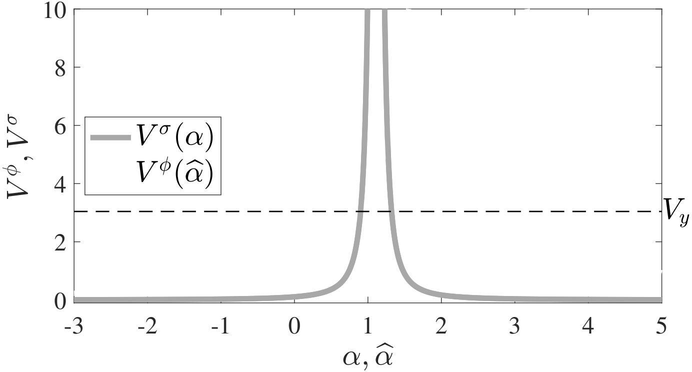
- Two configurations depending on the value of σ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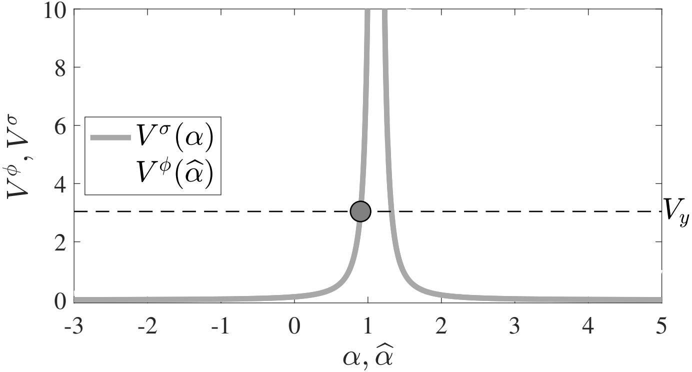
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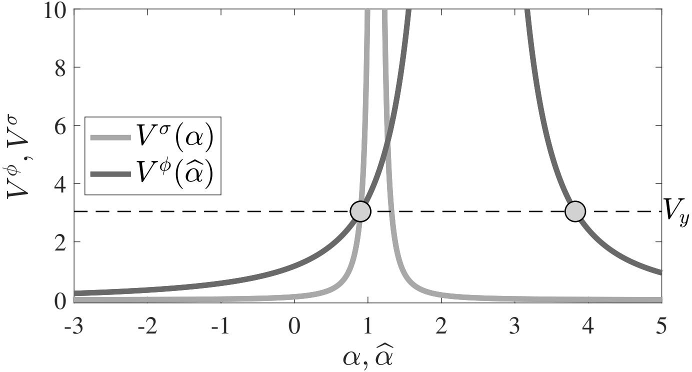

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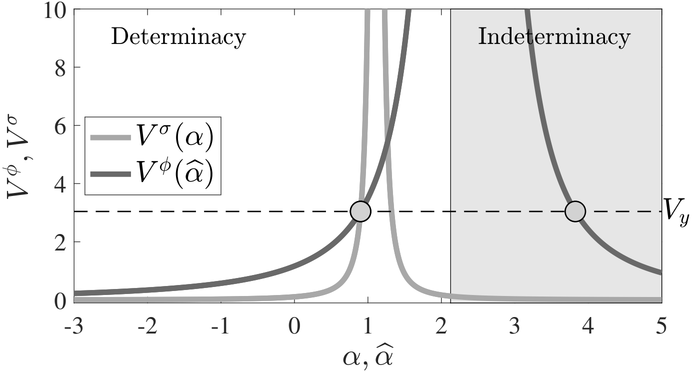
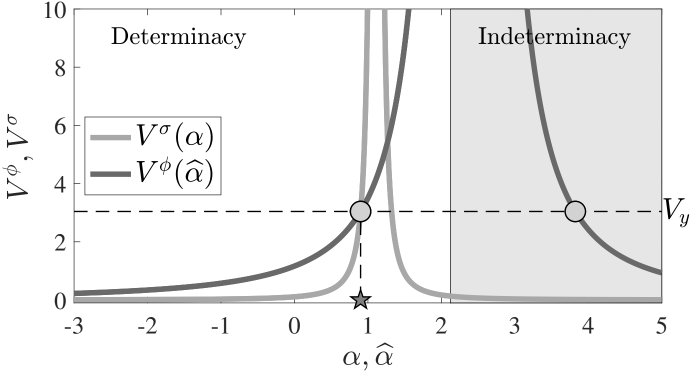
1. Abstract Approach
First Configuration




Determinacy
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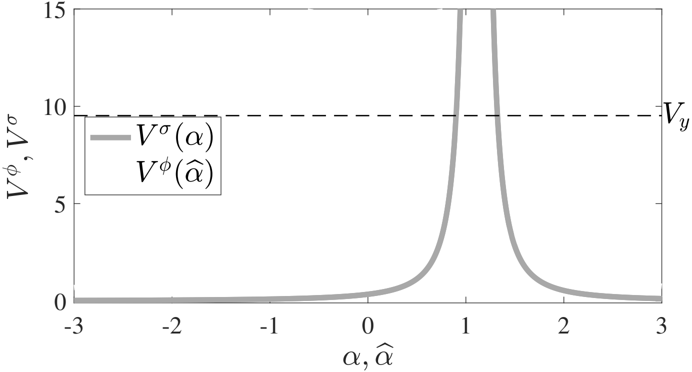
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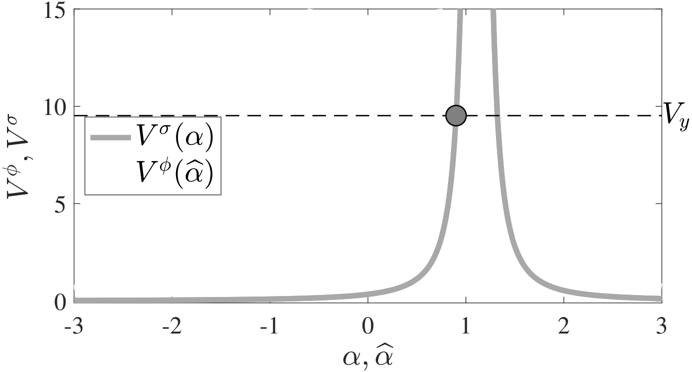
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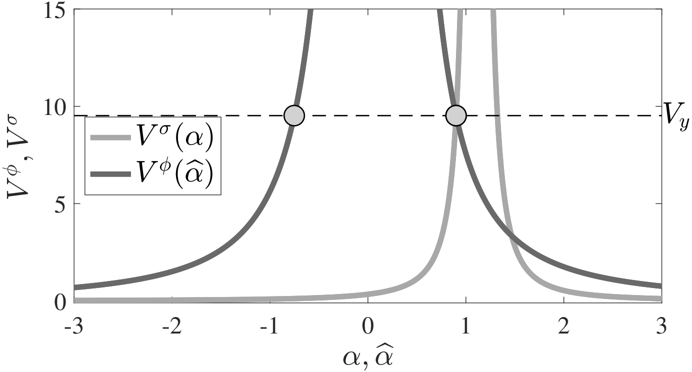
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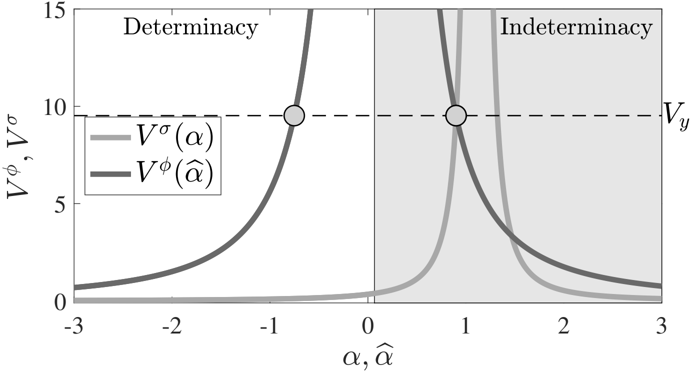
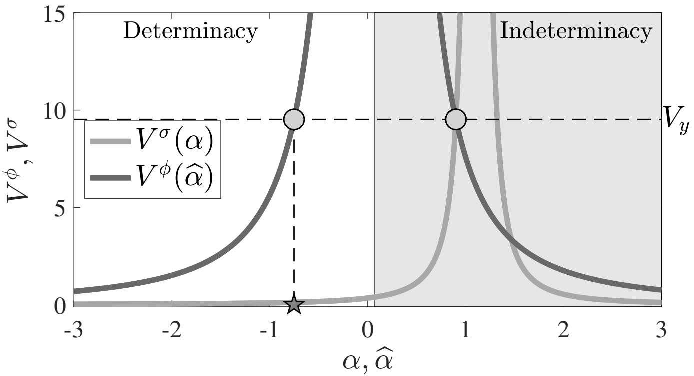
1. Abstract Approach
Second Configuration



1. Abstract Approach

Second Configuration



- When the Econometrician wrongly assumes that the DGP is a state-dependent rule: no bias.
- When the Econometrician wrongly assumes that the DGP is a feedback rule: possible bias depending on the value of σ
  - $\sigma \rightsquigarrow \phi^E$ that is in the determinacy zone: then all is good
  - $\sigma \rightsquigarrow \phi^E$ that is NOT in the determinacy zone: then biased estimation of α

- As α is just identified, the fit of the model is the same, even in the configuration in which $\widehat{\alpha}$ is biased.
- The problem here is that estimation, by keeping only determinate solutions, might bias the estimator.
- This is not the standard *misspecification bias* (see later)
- Let's call it a *determinacy bias*

- Assume now that we observe all the variables and estimate all the model parameters, when the state-dependent rule is the DGP, and in the two configurations (the feedback model with ϕ^E is determinate or not).
- Hard to see by hand what will be biased and how.
- Do Monte Carlo analysis and Max Likelihood with `Dynare` (which discards indeterminacy in estimation)

1. Abstract Approach

Estimation of a feedback rule model when the DGP is a state-dependent rule model

When the equivalent feedback model is determinate					
	$\sigma > -\frac{1+\alpha}{2\alpha\beta}$				
	α	β	ρ	σ	ϕ^E
DGP	.99	.2	.9	-4.31	-3.38

Notes: in this table we report the mean of the point estimates over the 100,000 simulations of length 300, with the standard deviation of the point estimates over the 100,000 simulations between parenthesis.

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	(.18)	(.05)	(.01)	–	(.02)

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When the equivalent feedback model is not determinate					
$\sigma < -\frac{1+\alpha}{2\alpha\beta}$					
	α	β	ρ	σ	ϕ^E
DGP	.99	.2	.9	-6.31	2.63

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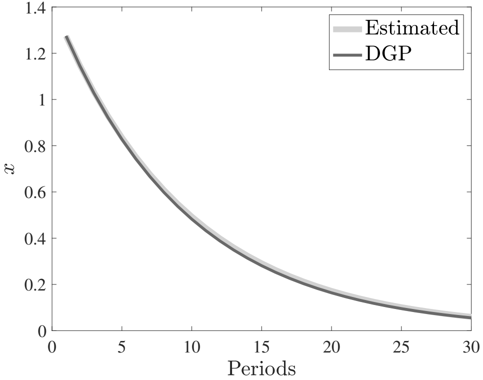
	α	β	ρ	σ	ϕ^E
DGP	.99	.2	.9	-6.31	2.63
Estimation	-.06	.24	.90	–	2.61
	(.09)	(.03)	(.01)	–	(.01)

Notes: in this table we report the mean of the point estimates over the 100,000 simulations of length 300, with the standard deviation of the point estimates over the 100,000 simulations between parenthesis.

1. Abstract Approach

DGP and estimated impulse responses to a s shock

Determinate equivalent feedback model

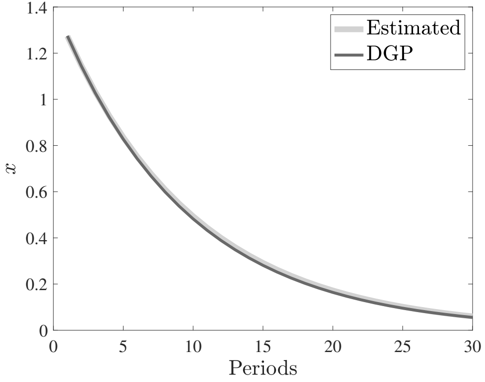


Indeterminate equivalent feedback model

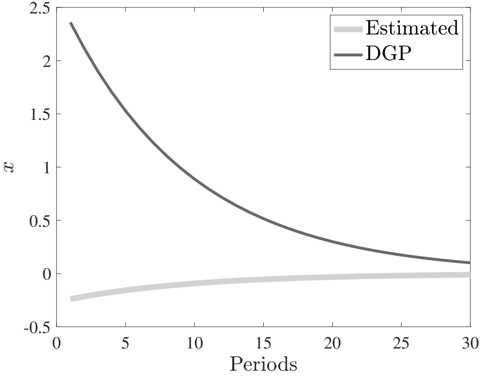
1. Abstract Approach

DGP and estimated impulse responses to a *s* shock

Determinate equivalent feedback model



Indeterminate equivalent feedback model



- Misspecification bias is well understood.
- Can be easily seen in a static version of the previous model.
- Variable of interest

$$y_t = \beta i_t + s_{1t} + \gamma s_{2t},$$

  (s_1, s_2) are two iid unit variance shocks.
- y is a policy variable that helps controlling x.
  - × Feedback rule: $i_t = \phi y_t$
  - × state-dependent rule: $i_t = \sigma_1 s_{1t} + \sigma_2 s_{2t}$
- Feedback rule constrains y to react to s_1 and s_2 in proportion 1 to γ.
- Assume we know β, the shock variances, and aim at estimating γ.

- If the DGP has a feedback rule, one can show that, when the econometrician assumes a state-dependent rule, γ is identified and its OLS estimate is

$$\widehat{\gamma} = \gamma$$

- If the DGP has a state-dependent rule, one can show that, when the econometrician assumes a feedback rule, γ is identified and its OLS estimate is

$$\widehat{\gamma} = \left(1 - \frac{\beta}{2}(\sigma_1 + \sigma_2)\right)\gamma + \beta\sigma_2\left(1 - \frac{\beta}{2}(\sigma_1 + \sigma_2)\right)$$

- The estimation of γ will be biased (unless $\sigma_2 = \gamma\sigma_1$)

1. Abstract Approach

- Are there evidence of determinacy and specification bias when estimating DSGE-like models?

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

- The model is the basic forward looking three-equation NK model (Galí)

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, \qquad \text{(Euler Equation)}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + \mu_t, \qquad \text{(Phillips Curve)}$$

- Policy rule:
  - The model is generally closed with a Taylor rule (TR) of the type:

$$i_t = \phi_y y_t + \phi_\pi \pi_t + \nu_t, \qquad \text{(Taylor Rule)}$$

  -

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$$r_t = -E_t \pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, \qquad \text{(Taylor Rule)}$$

  -

2. A Simple Three-Equation Estimated Model
Model

- The model is the basic forward looking three-equation NK model (Galí)

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r \underbrace{(i_t - E_t[\pi_{t+1}])}_{r_t} + d_t, \qquad \text{(Euler Equation)}$$

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- Policy rule:
  - The model is generally closed with a Taylor rule (TR) of the type:

$$r_t = -E_t \pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, \qquad \text{(Taylor Rule)}$$

  - We alternatively close the model with the following state-dependent rule that we call "Real rate Rule" (RR):

$$r_t = \phi_d d_t + \phi_\mu \mu_t + \widetilde{\nu}_t. \qquad \text{(Real rate Rule)}$$

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r r_t + d_t, \qquad \text{(Euler Equation)}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + \mu_t, \qquad \text{(Phillips Curve)}$$

$$r_t = \begin{cases} -E_t \pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, & \text{(Taylor Rule)} \quad \text{(feedback)} \\ \phi_d d_t + \phi_\mu \mu_t + \widetilde{\nu}_t. & \text{(Real rate Rule)} \quad \text{(state-dependent)} \end{cases}$$

- Remark 1: $|\alpha_y| < 1 \rightsquigarrow$ always determinacy under RR (and α_y can be arbitrarily close to 1)
- Remark 2: We are in the configuration of the abstract example (under RR, one can compute the Equivalent TR and reciprocally)
- Remark 3: RR encompasses TR.

$$r_t = \begin{cases} -E_t\pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, & \text{(Taylor Rule) (feedback)} \\ \phi_d d_t + \phi_\mu \mu_t + \widetilde{\nu}_t. & \text{(Real rate Rule) (state-dependent)} \end{cases}$$

- Remark 4: A real interest rule might be a realistic description of the Fed behavior.
- Realism matters for interpretation, but not for estimation
- State-dependent Monetary policy is just choosing allocations along Euler Equation and Phillips Curve as a function of the state of the economy
- Same deep parameters are estimated if monetary policy is

$$y_t = \omega_d d_t + \omega_\mu \mu_t + \widehat{\nu}_t \qquad \text{(Another Monetary Policy Rule)}$$

2. A Simple Three-Equation Estimated Model
Estimation

- US data, `1959Q1-2019Q4`.
- Output gap from the CBO, log difference of the CPI for inflation, Federal Funds rate for i.
- The shadow Federal Funds rate from Wu and Xia (2016) is used from 2009 onwards - the period when the zero lower bound might be a binding constraint.
- Calibrated: $\beta = .99$ $\alpha_r = 1$ (log utility) and $\alpha_y = .999$ (quasi no Euler-discounting)
- Bayesian estimation

Estimated Slope of the Phillips Curve, Taylor rule (TR) *versus* Real rate Rule (RR)

Taylor Rule		Real rate Rule	
κ	0.68	κ	0.006
	(0.06)		(0.001)

Estimated and Implied Policy Parameters

Taylor Rule		Real rate Rule	
ϕ_π	1.77		
ϕ_y	-0.01		
		ϕ_d	0.97
		ϕ_μ	-0.46

Estimated and Implied Policy Parameters

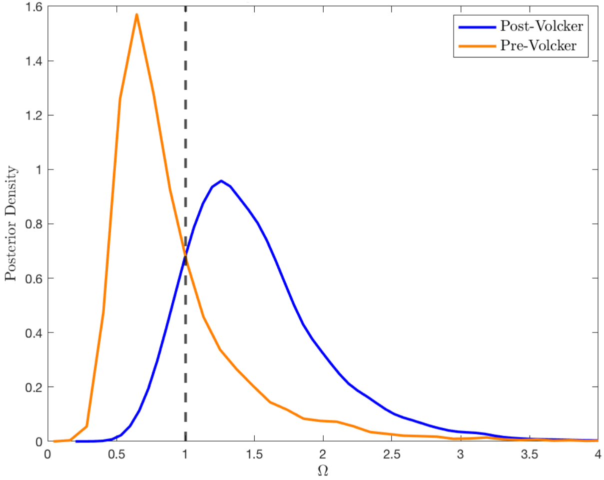
Taylor Rule		Real rate Rule	
ϕ_π	1.77	ϕ_π^E	-0.24
ϕ_y	-0.01	ϕ_y^E	0.68
		ϕ_d	0.97
		ϕ_μ	-0.46

There is a determinacy bias

- Clarida, Galí and Gertler [2000]:
  - Estimate a Taylor rule on Pre- versus Post-Volcker data.
  - Find that Pre-Volcker Taylor rule leads to indeterminacy once put in a "simple" NK model.
- Lubik and Schorfheide [2004]:
  - Full information estimation of a NK model with Taylor rule and allowing for sunspot solutions
  - Find that U.S. monetary policy post-1982 is consistent with determinacy, whereas the pre-Volcker policy is not.

- We estimate separately the model over the two sub-periods with a Real Rate rule.
- We find that the equivalent Taylor rule creates indeterminacy pre-Volcker, not post-Volcker

Is the Equivalent Taylor rule Determinate?
(posterior distribution of Ω, $\Omega > 1 \rightsquigarrow$ indeterminacy)



- Estimating with a Taylor Rule
  - × We never find indeterminacy (by construction, as this is ruled out by the estimation)
  - × But the Phillips curve κ is unstable between the two periods: $\kappa = .22 \rightsquigarrow \kappa = .75$ (determinacy bias)
- Estimating with a state-dependent Real Rate rule
  - × We find no flattening of the Phillips curve: $\kappa = .003 \rightsquigarrow \kappa = .0015$
  - × The real rate rule becomes more aggressive for demand shocks, less for supply ones.

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

- Take a richer model with habit persistence and a hybrid Phillips curve.
-

$$i_t = \begin{cases} \phi_y y_t + \phi_\pi \pi_t + \nu_t, & \text{(Taylor Rule)} \\ E_t[\pi_{t+1}] + \phi_{y_{-1}} y_{t-1} + \phi_{\pi_{-1}} \pi_{t-1} + \phi_d d_t + \phi_\mu \mu_t + \nu_t. & \text{(Real rate Rule)} \end{cases}$$

- We obtain the same results:
  - × The implied TR is in the indeterminacy zone
  - × Phillips curve is flatter with the RR: $\kappa = 0.004$ instead of 0.67.

- Take a simple-to-solve HANK model, as written by Broer, Harbo Hansen, Krusell and Öberg [2020].
- It is a Huggett-Aiyagari model with sticky wages and some tricks to make computation easier.
- We obtain the same results:
  - × The implied TR is in the indeterminacy zone
  - × Phillips curves are much flatter with the RR: $\kappa_P = 0.003$ instead of 0.72, $\kappa_W = 0.03$ instead of 0.15

- Large model with 7 shocks, 36 estimated coefficients, 19 state variables
- 7 observable series, Bayesian estimation
- Estimated Taylor Rule in S&W:

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y (y_t - y_t^f)\right) + \phi_{\Delta y}\left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\right) + \varepsilon_t^i$$

- The Real rate Rule:

$$\begin{aligned} r_t - E_t \pi_{t+1} &= \sigma_a \varepsilon_t^a + \sigma_b \varepsilon_t^b + \sigma_g \varepsilon_t^g + \sigma_{i^s} \varepsilon_t^i + \sigma_{p^{inf}} \varepsilon_t^p + \sigma_{w^s} \varepsilon_t^w + \sigma_{\eta^w} \eta_{t-1}^w \\ &+ \sigma_{\eta^p} \eta_{t-1}^p + \sigma_{y^p} y_{t-1}^p + \sigma_y y_{t-1} + \sigma_r r_{t-1} + \sigma_{k^{p,s}} k_t^{p,s} + \sigma_{k^s} k_t^s \\ &+ \sigma_{c^p} c_{t-1}^p + \sigma_{i^p} i_{t-1}^p + \sigma_c c_{t-1} + \sigma_i i_{t-1} + \sigma_\pi \pi_{t-1} + \sigma_w w_{t-1} + \varepsilon_t^R \end{aligned}$$

- (Is the Central Bank information set realistic?)

- Let's focus on the slope of the Phillips Curve
- The slope of the Phillips curve at the posterior mean is .003 for the TR model $\rightsquigarrow$ there is almost no Phillips curve in Smets & Wouters (well known?)
- Slope is not significantly different in the RR model.

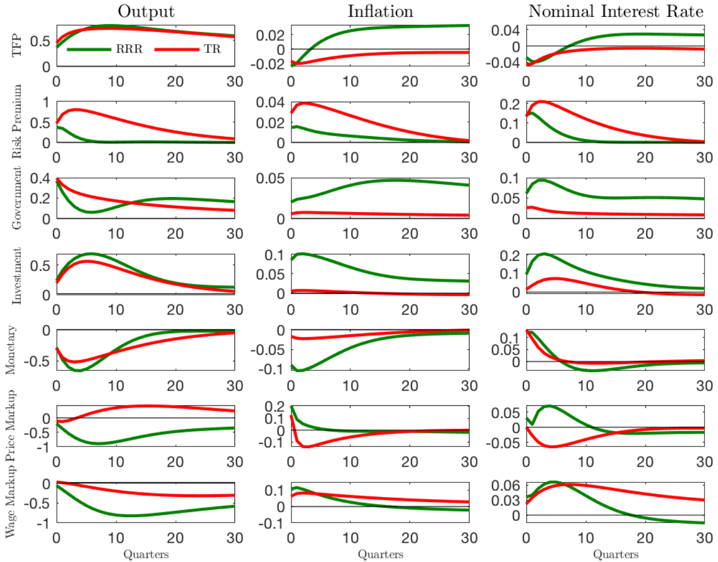
- No sign of determinacy bias with the SW extended Taylor rule.

	Implied Extended Taylor Rule	Estimated Extended Taylor Rule
ϕ_π	1.43	1.84
ϕ_y	0.00	0.11
ρ	0.84	0.87
$\phi_{\Delta y}$	0.17	0.25

- But evidence of misspecification bias: impulse responses and variance decomposition are pretty different
- Reminder: the (state-dependent) real rate rule *encompasses* the Taylor rule.

3. Extensions

Smets & Wouters [2007]: TR and RR: Possible misspecification bias



Inflation Unconditional Variance Decomposition

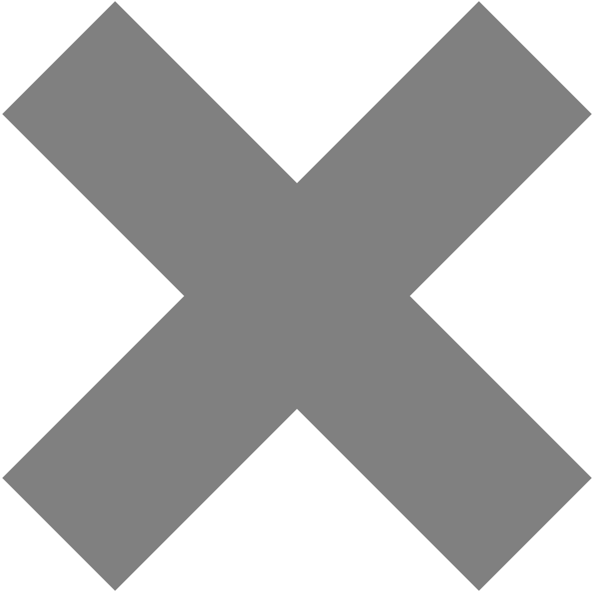
	Full real rate rule	Smets-Wouters
TFP		2
Risk Premium		6
Government Spending		1
Investment Technology		0
Monetary Policy		2
Price Markup		47
Wage Markup		41

Inflation Unconditional Variance Decomposition

	Full real rate rule	Smets-Wouters
TFP	16	2
Risk Premium	0	6
Government Spending	16	1
Investment Technology	21	0
Monetary Policy	13	2
Price Markup	14	47
Wage Markup	20	41

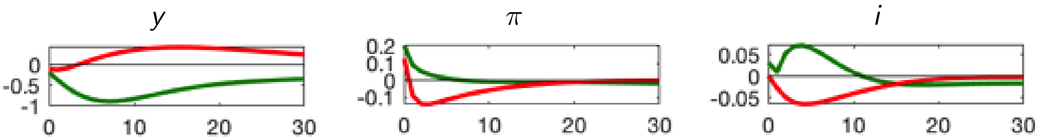
Conclusion

- Specification of the monetary policy rule matters big time
- Evidence of both determinacy and misspecification bias.
- I would advise practitioners to use a state-dependent monetary policy rules, or at least check if results change a lot with such a rule.



Smets & Wouters [2007]: TR and RR: Possible misspecification bias

Response to a price markup shock



(red for the Taylor Rule, green for the Real Rate Rule

Output Unconditional Variance Decomposition

	Full real rate rule	Smets-Wouters
TFP	38	51
Risk Premium	1	16
Government Spending	2	2
Investment Technology	8	8
Monetary Policy	4	6
Price Markup	19	7
Wage Markup	30	9