Some Inference Perils of Imposing a Taylor Rule

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Motivation

- Sticky prices models do not restrict much equilibrium allocations.
- What matters is the bundle $\{$sticky prices , monetary policy$\}$.

Motivation

An introductory example

- Basic 3-equation New-Keynesian model

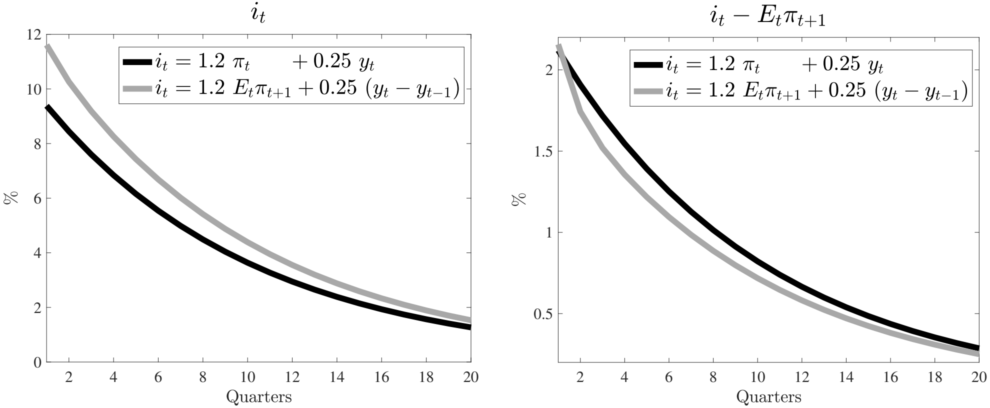
$$y_t = E_t[y_{t+1}] - 1 \times (i_t - E_t[\pi_{t+1}]) + d_t, \qquad \text{(Euler Equation)}$$
$$\pi_t = 0.99 \times E_t[\pi_{t+1}] + 0.1 \times y_t + \mu_t. \qquad \text{(Phillips Curve)}$$

- Two "TAYLOR rules" :

$$\begin{aligned} i_t &= 1.2 \times \pi_t &+& \ 0.25 \times y_t, \\ i_t &= 1.2 \times E_t\pi_{t+1} &+& \ 0.25 \times (y_t - y_{t-1}). \end{aligned}$$

- Shock $d_0 = 2$, $\mu_0 = 1$ (persistence .9 and .9).

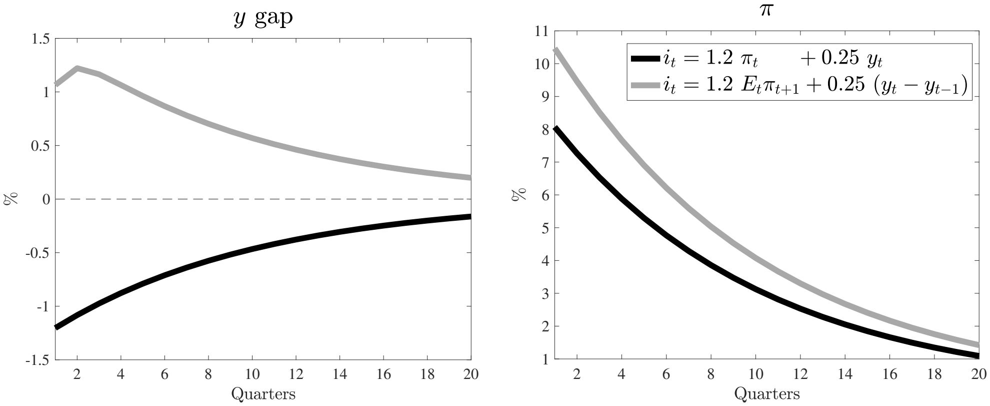
Response to a 2 s.d. demand shock *and* a 1 s.d. markup shock – Policy Instruments



Motivation

An introductory example

Response to a 2 s.d. demand shock *and* a 1 s.d. markup shock – Output and inflation



Motivation

- From that example, we see that the specification of the monetary policy rule matters big time.

- A lot of microfoundation efforts in estimated DSGE ...
- but not much thoughts[1] on how to specify the monetary policy rule ...
- *as if it should not matter*...
- although it does matter.

[1] It is interesting to look back and see when, how and why TAYLOR rules were introduced in estimated models.

What we do?

- Explore consequences of assuming (possibly wrongly) a Taylor rule when estimating NK models.
- Find it biaises estimated deep parameters:
  - × *determination bias* in small models,
  - × *misspecification bias* in larger models.
- Solution: use an agnostic (linear, minimal) *state* monetary policy rule (that maps the state of the economy into an equilibrium allocation).
- (modest) Contribution: show that the monetary policy rule specification does matter *in practice* when estimating DSGEs, and to propose an alternative.

Message

- It is a bad idea to estimate a NK DSGE assuming monetary authorities follow a TAYLOR rule.

Literature

- There is a literature on optimal monetary policy – not our concern here.

Important remark

- We follow the practice of restricting to determinate equilibria.
- In estimation, one identifying restriction is equilibrium determinacy.

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions (Today Smets & Wouters [2007])

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

1. Abstract Approach

- Goal: show the possibility of a determinacy bias and a misspecification bias when using a Taylor rule
- The distinctive property of the Taylor rule is that it is a *feedback rule*.
- Start with the determinacy bias.

$$y_t = \alpha E_t y_{t+1} + \beta i_t + s_t, \qquad 0 \leq \alpha < 1 \text{ and } \beta > 0,$$
$$s_t = \rho s_{t-1} + \varepsilon_t, \qquad 0 \leq \rho < 1 \text{ and } V(\varepsilon) = 1.$$

- i is a policy variable that helps controlling y:
  - feedback rule (ϕ for ϕ*eedback*) ("TAYLOR rule"):

    $$i_t = \phi y_t,$$

  - (linear minimal) state rule (σ for σ*tate*):

    $$i_t = \sigma s_t.$$

1. Abstract Approach

Solution with a feedback rule $\quad i_t = \phi y_t$

- $y_t = \alpha E_t y_{t+1} + \beta \underbrace{i_t}_{\phi y_t} + s_t.$
- Solving forward:

$$y_t = \frac{1}{1-\beta\phi}\left(\sum_{j=0}^{\infty}\left(\frac{\alpha\rho}{1-\beta\phi}\right)^j\right)s_t.$$

- Converges for any persistence parameter $0 \leq \rho < 1$ if $\left|\frac{\alpha}{1-\beta\phi}\right| < 1$.
- Therefore, the restriction on policy to have determinacy ("TAYLOR principle") is :

$$\phi \notin \left]\frac{1-\alpha}{\beta}, \frac{1+\alpha}{\beta}\right[,$$

- and solution is:

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\, s_t.$$

1. Abstract Approach

Solution with a state rule $i_t = \sigma s_t$

- $y_t = \alpha E_t y_{t+1} + \beta \underbrace{i_t}_{\sigma s_t} + s_t.$
- Solving forward:

$$y_t = (1 + \beta\sigma) \left(\sum_{j=0}^{\infty} (\alpha\rho)^j \right) s_t.$$

- *The sum converges for any policy choice σ,*
- and solution is

$$y_t = \frac{1 + \beta\sigma}{1 - \alpha\rho} \; s_t.$$

Feedback rule

$$i_t = \phi \, y_t$$

State rule

$$i_t = \sigma \, s_t$$

Solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho} \, s_t$$

Solution

$$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \, s_t$$

DGP = Feedback rule

$$i_t = \phi\, y_t$$

Solution

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\, s_t$$

State rule

$$i_t = \sigma^E\, s_t$$

Solution

$$y_t = \frac{1+\beta\sigma^E}{1-\alpha\rho}\, s_t$$

– **Equivalent state rule:** σ^E *iif*

$$\frac{1}{1-\beta\phi-\alpha\rho} = \frac{1+\beta\sigma^E}{1-\alpha\rho} \iff \sigma^E = \frac{\phi}{1-\beta\phi-\alpha\rho}.$$

– *Any feedback rule allocation can be replicated by a state rule.*

1. Abstract Approach

Equivalent feedback rule ϕ^E

Feedback rule

$i_t = \phi^E \; y_t$

DGP = State rule

$i_t = \sigma \; s_t$

Solution

$y_t = \frac{1}{1-\beta\phi^E-\alpha\rho} \; s_t$

Solution

$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \; s_t$

– **Equivalent feedback rule:** ϕ^E must solve

$$\frac{1}{1-\beta\phi^E-\alpha\rho} = \frac{1+\beta\sigma}{1-\alpha\rho} \iff \phi^E = \frac{\sigma(1-\alpha\rho)}{1+\beta\sigma}.$$

Feedback rule

$i_t = \phi^E \; y_t$

DGP = State rule

$i_t = \sigma \; s_t$

Solution

$y_t = \frac{1}{1-\beta\phi^E-\alpha\rho} \; s_t$

Solution

$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \; s_t$

– **Equivalent feedback rule:** ϕ^E must solve

$$\frac{1}{1-\beta\phi^E-\alpha\rho} = \frac{1+\beta\sigma}{1-\alpha\rho} \Longleftrightarrow \phi^E = \frac{\sigma(1-\alpha\rho)}{1+\beta\sigma} \qquad \overset{?}{\notin} \quad \left]\frac{1-\alpha}{\beta}, \frac{1+\alpha}{\beta}\right[.$$

Equivalent feedback rule ϕ^E

Feedback rule

$i_t = \phi^E \; y_t$

DGP = State rule

$i_t = \sigma \; s_t$

Solution

$y_t = \frac{1}{1-\beta\phi^E-\alpha\rho} \; s_t$

Solution

$y_t = \frac{\boldsymbol{1+\beta\sigma}}{\boldsymbol{1-\alpha\rho}} \; s_t$

- **Equivalent feedback rule:** ϕ^E must solve

$$\frac{1}{1-\beta\phi^E-\alpha\rho} = \frac{1+\beta\sigma}{1-\alpha\rho} \Longleftrightarrow \phi^E = \frac{\sigma(1-\alpha\rho)}{1+\beta\sigma} \qquad \overset{?}{\notin} \quad \left]\frac{1-\alpha}{\beta}, \frac{1+\alpha}{\beta}\right[.$$

× If $\sigma > -\frac{1+\alpha}{2\alpha\beta}$, then ϕ^E will satisfy the determinacy condition.

× But if $\sigma < -\frac{1+\alpha}{2\alpha\beta}$, there is no determinate feedback model that can reproduce the state rule allocations $\rightsquigarrow$ *No equivalence.*

- Wrongly assuming a feedback rule can create a bias in estimation of deep parameters.
- This can be shown analytically in our abstract model.

- Here an extreme example for which the bias can be analytically computed.
- Only y is observed.
- All parameters are known $(\beta,\ \rho,\ \phi$ or σ^E, σ or $\phi^E)$ but α.

$$y_t = \alpha E_t y_{t+1} + \beta i_t + s_t.$$

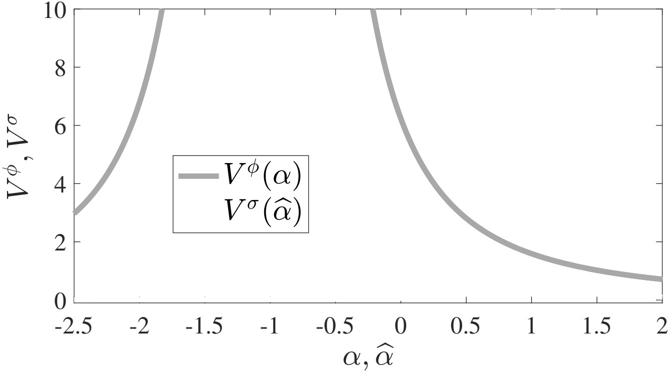
- α can be estimated (ML) by matching the observed variance of y: V_y.
- Assume true $\alpha = .9$.

– Assume the DGP is the feedback rule (ϕ) but the econometrician believes it is a state rule model.

State rule	**Feedback rule**
$i_t = \sigma^E \, s_t$	$i_t = \phi \, y_t$
Solution	**Solution**
$y_t = \frac{1+\beta\sigma^E}{1-\widehat{\alpha}\rho} \, s_t$	$y_t = \frac{1}{1-\beta\phi-\alpha\rho} \, s_t$
$\boldsymbol{V}^\sigma(\widehat{\alpha})$	$\boldsymbol{V}^\phi(\alpha)$
$\left(\frac{1+\beta\sigma^E}{1-\widehat{\alpha}\rho}\right)^2 \frac{1}{1-\rho^2}$	$\frac{1}{(1-\beta\phi-\alpha\rho)^2} \, \frac{1}{1-\rho^2}$

1. Abstract Approach

DGP is ϕ, econometrician believes it is σ



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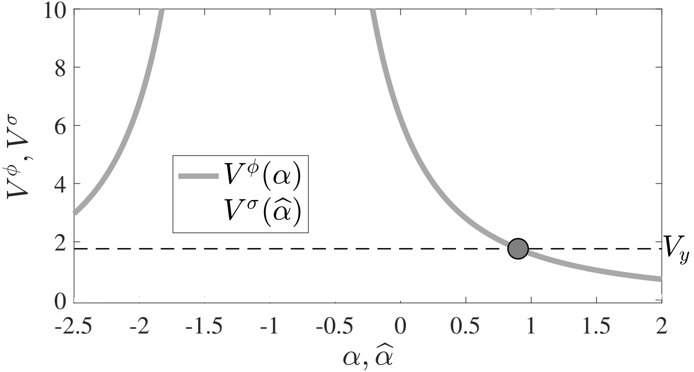

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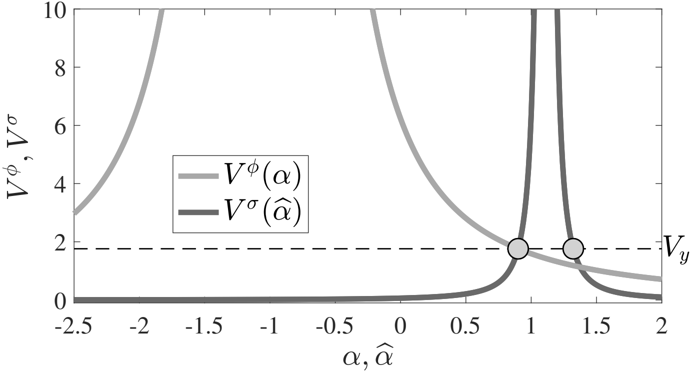

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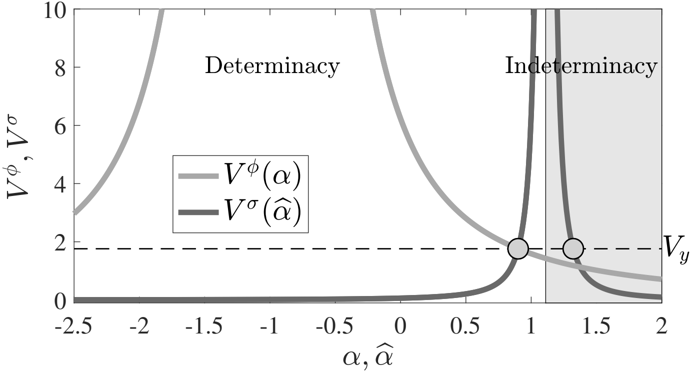

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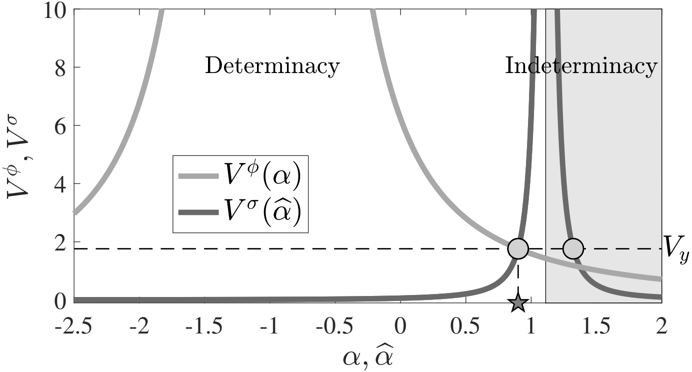
DGP is ϕ, econometrician believes it is σ

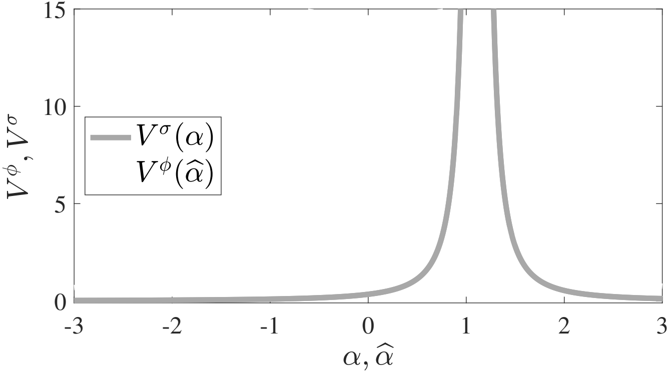


- Assume now that the DGP is the state rule (σ) but the econometrician believes it is a feedback rule model.
- If $\sigma \rightsquigarrow \phi^E$ that is not in the determinacy zone: biased estimation of α.

Feedback rule	**State rule**
$i_t = \phi^E \; y_t$	$i_t = \sigma \; s_t$
Solution	**Solution**
$y_t = \frac{1}{1-\beta\phi^E-\widehat{\alpha}\rho} \; s_t$	$y_t = \frac{1+\beta\sigma}{1-\alpha\rho} \; s_t$
$\boldsymbol{V}^{\phi}(\widehat{\alpha})$	$\boldsymbol{V}^{\sigma}(\alpha)$
$\frac{1}{(1-\beta\phi^E-\widehat{\alpha}\rho)^2} \; \frac{1}{1-\rho^2}$	$\left(\frac{1+\beta\sigma}{1-\alpha\rho}\right)^2 \frac{1}{1-\rho^2}$

1. Abstract Approach

DGP is σ, econometrician believes it is ϕ – Determinacy bias



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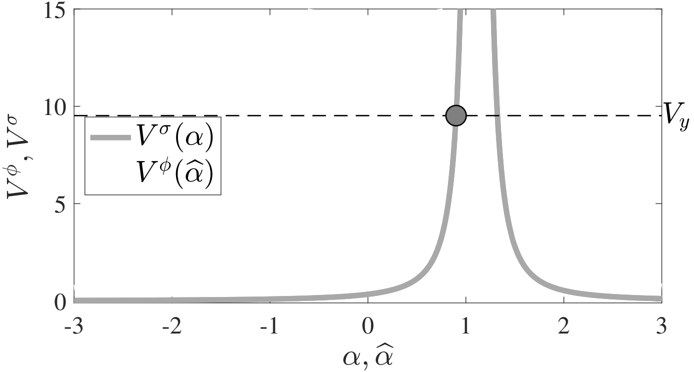

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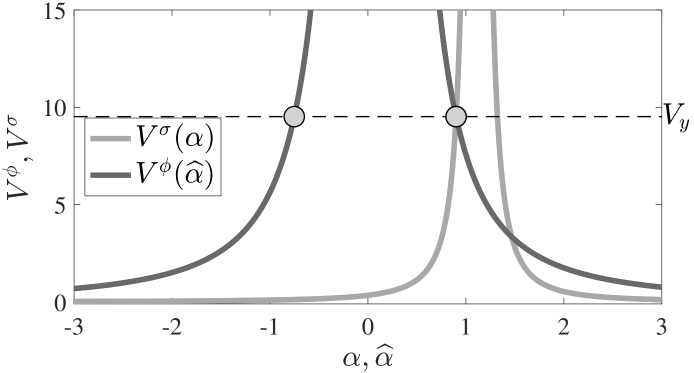

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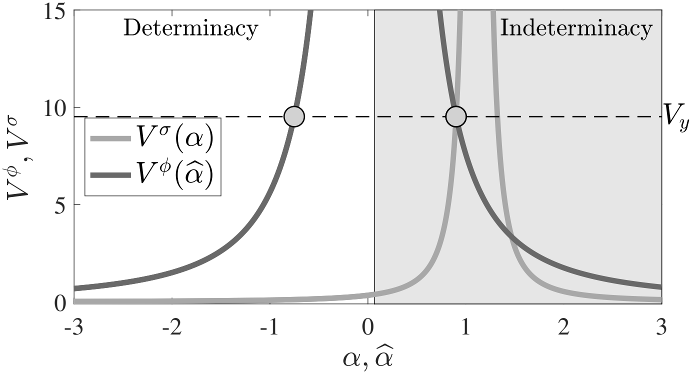

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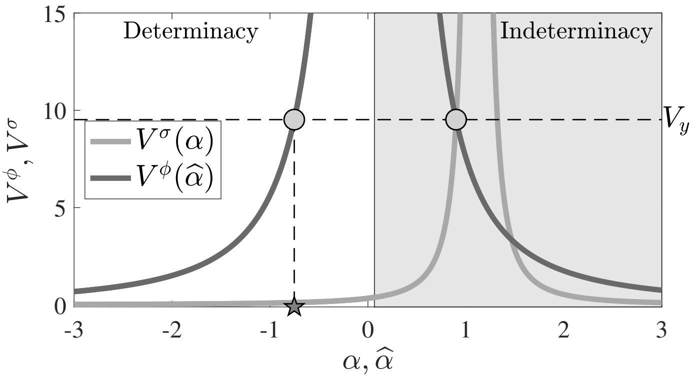


- As α is just identified, the fit of the model is the same, even if $\widehat{\alpha}$ is biased.
- This is not the standard *misspecification bias* (see next).
- Let's call it a *determinacy bias*.

- Static model

$$y_t = \beta i_t + s_{1t} + \gamma \, s_{2t},$$

- Rules:

Feedback rule	State rule
$i_t = \phi y_t + \nu_t$	$i_t = \sigma_1 s_{1t} + \sigma_2 s_{2t} + \nu_t$

- Bias in estimating γ for β and shock variances known if feedback rule wrongly assumed.

1. Abstract Approach

- Are there evidence of determinacy and specification bias when estimating DSGE-like models?
- Yes

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r r_t + d_t, \qquad \text{(Euler Equation)}$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + \mu_t, \qquad \text{(Phillips Curve)}$$

$$r_t = \begin{cases} -E_t\pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, & \text{(Taylor Rule)} \quad (\phi\textbf{eedback}) \\ \sigma_d d_t + \sigma_\mu \mu_t + \widetilde{\nu}_t. & \text{(Real rate Rule)} \quad (\sigma\textbf{tate}) \end{cases}$$

- Remark 1: $|\alpha_y| < 1 \rightsquigarrow$ always determinacy under RR (and α_y can be arbitrarily close to 1)
- Remark 2: Same deep parameters are estimated if monetary policy is

$$y_t = \sigma'_d d_t + \sigma'_\mu \mu_t + \widehat{\nu}_t \qquad \text{(Another Monetary Policy State Rule)}$$

- US data, `1959Q1–2019Q4`.
- Output gap from the CBO, log difference of the CPI for inflation, Federal Funds rate for i.
- The shadow Federal Funds rate from Wu and Xia (2016) is used from 2009 onwards - the period when the zero lower bound might be a binding constraint.

- Bayesian estimation

$$y_t = 0.999 \times E_t[y_{t+1}] - 1 \times r_t + d_t, \qquad \text{(Euler Equation)}$$

$$\pi_t = 0.99 \times E_t[\pi_{t+1}] + \kappa y_t + \mu_t, \qquad \text{(Phillips Curve)}$$

$$r_t = \begin{cases} -E_t \pi_{t+1} + \phi_y y_t + \phi_\pi \pi_t + \nu_t, & \text{(Taylor Rule)} \\ \sigma_d d_t + \sigma_\mu \mu_t + \widetilde{\nu}_t. & \text{(State Rule)} \end{cases}$$

$$d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{dt} \qquad \text{(Demand shock)}$$

$$\mu_t = \rho_\mu d_{t-1} + \sigma_\mu \varepsilon_{\mu t} \qquad \text{(Supply shock)}$$

$$\nu_t = \rho_\nu d_{t-1} + \sigma_\nu \varepsilon_{\nu t} \qquad \text{(Monetary shock)}$$

Estimated Slope of the Phillips Curve, Taylor rule *versus* State Rule

Taylor Rule		State Rule	
κ	0.68	κ	0.006
	(0.06)		(0.001)

– Here the State Rule is expressed as a Real rate Rule.

Estimated and Implied Policy Parameters

Taylor Rule		Real rate Rule	
ϕ_π	1.77		
ϕ_y	-0.01		
		σ_d	0.97
		σ_μ	-0.46

Estimated and Implied Policy Parameters

Taylor Rule		Real rate Rule	
ϕ_π	1.77	ϕ_π^E	-0.24
ϕ_y	-0.01	ϕ_y^E	0.68
		σ_d	0.97
		σ_μ	-0.46

The equivalent Taylor rule does not satisfy Taylor principle $\rightsquigarrow$ There is a determinacy bias.

Roadmap

1. Abstract Approach
2. A Simple Three-Equation Estimated Model
3. Extensions (Today Smets & Wouters [2007])

- Large model with 7 shocks, 36 estimated coefficients, 19 state variables
- 7 observable series, Bayesian estimation
- Estimated TAYLOR Rule in S&W:

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y (y_t - y_t^f)\right) + \phi_{\Delta y}\left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\right) + \varepsilon_t^i$$

- The Real rate Rule (The State Rule):

$$\begin{aligned} r_t - E_t \pi_{t+1} &= \sigma_a \varepsilon_t^a + \sigma_b \varepsilon_t^b + \sigma_g \varepsilon_t^g + \sigma_{i^s} \varepsilon_t^i + \sigma_{p^{inf}} \varepsilon_t^p + \sigma_{w^s} \varepsilon_t^w + \sigma_{\eta^w} \eta_{t-1}^w \\ &+ \sigma_{\eta^p} \eta_{t-1}^p + \sigma_{y^p} y_{t-1}^p + \sigma_y y_{t-1} + \sigma_r r_{t-1} + \sigma_{k^{p,s}} k_t^{p,s} + \sigma_{k^s} k_t^s \\ &+ \sigma_{c^p} c_{t-1}^p + \sigma_{i^p} i_{t-1}^p + \sigma_c c_{t-1} + \sigma_i i_{t-1} + \sigma_\pi \pi_{t-1} + \sigma_w w_{t-1} + \varepsilon_t^R \end{aligned}$$

- Is the Central Bank information set realistic? $\rightsquigarrow$ If not, some σ_x will be zero.

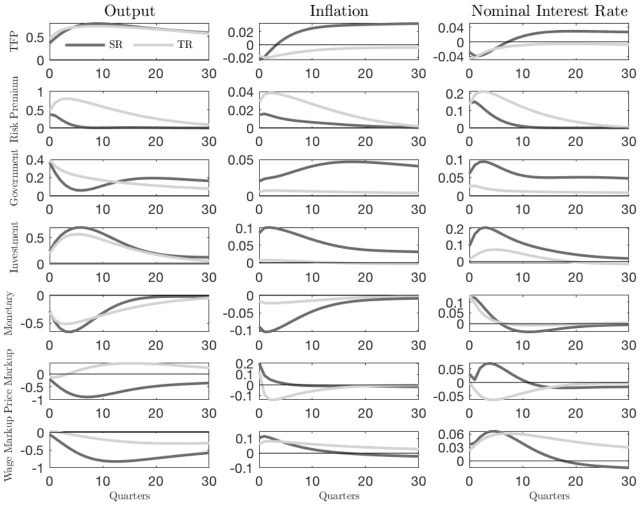
– No sign of determinacy bias with the SW extended Taylor rule.

	S&W Taylor Rule	Implied Taylor Rule
ϕ_π	1.84	1.43
ϕ_y	0.11	0.00
ρ	0.87	0.84
$\phi_{\Delta y}$	0.25	0.17

- But evidence of misspecification bias:
- Parameters posterior distributions are pretty different $\rightsquigarrow$ impulse responses and variance decomposition are pretty different.
- Reminder: the (state) real rate rule *encompasses* the Taylor rule.

3. Extensions

Smets & Wouters [2007]: Taylor Rule (TR) and State Rule (SR): misspecification bias



Inflation Unconditional Variance Decomposition (in %)

	State rule	Smets-Wouters
TFP		2
Risk Premium		6
Government Spending		1
Investment Technology		0
Monetary Policy		2
Price Markup		47
Wage Markup		41

Inflation Unconditional Variance Decomposition (in %)

	State rule	Smets-Wouters
TFP	16	2
Risk Premium	0	6
Government Spending	16	1
Investment Technology	21	0
Monetary Policy	13	2
Price Markup	14	47
Wage Markup	20	41

Conclusion

- Specification of the monetary policy rule matters big time
- Evidence of both determinacy and misspecification bias.
- We recommend to use a state monetary policy rules,
- At least check if results change a lot with a state rule.